Numerical modeling of the mean exchange through the Strait of Gibraltar

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Abstract. The three-dimensional Princeton Ocean Model is used to investigate the mean flow and the hydraulics regime in the Strait of Gibraltar. The model makes use of a coast following curvilinear orthogonal grid, that include the Gulf of Cadiz and the Alboran Sea with very high resolution in the strait (» 500 m). A lock-exchange initial condition is used: the western part of the model domain is filled with Atlantic water, whereas the eastern part with Mediterranean water. A conservative, non dispersive numerical scheme for the horizontal tracers advection has been implemented to simulate the free flow adjustment for the lock-exchange initial condition. Predicted velocities, interface depth, water and salinity transports are comparable with observed data. Model results describe in detail circulation of the Mediterranean outflow and demonstrate the presence of a mixing layer between the Atlantic and Mediterranean water. Hydraulic regime is analyzed calculating the Composite Froude number for three layers within the strait. Results indicate the presence of a hydraulic control at Camarinal Sill and in the northern part of Tarifa Narrows. In order to understand if the Strait of Gibraltar is in the submaximal or maximal regime we have formulated a cross-strait mean Composite Froude number. This formulation allowed us to limit the values of the composite Froude number in the range 1:2 to 1:5 at Camarinal Sill and between 0:4 to 0:8 at Tarifa Narrows. We have concluded that the mean exchange simulated by the model is in submaximal regime.
1. Introduction

Since ancient times the Fretum Gaditanum (the Roman name of the Strait of Gibraltar) has fascinated and captured mankind's imagination: for a long time Hercules' pillars, the two promontories of Calpe (now Gibraltar) in Spain and Abila (now Jebel Sidi Moussa) in Morocco, were thought to be the extreme edges of the Earth. Nowadays the strait still attracts considerable interest as a relevant factor in the Mediterranean thermohaline circulation and the intermediate and deep water circulation of the North Atlantic (Reid, 1979) (Price et al., 1993).

The narrow and shallow Strait of Gibraltar connects the Atlantic Ocean with the Mediterranean Sea. It is about 60 K m long and about 20 K m wide with a minimum width of 15 K m near Tarifa and a shallow sill located near Camarinal (west of Tarifa) with a minimum depth of less than 300 m (Figure 1).

An excess of evaporation with respect to the freshwater input and the conservation of mass and salt in the Mediterranean Sea drive the mean circulation in the strait. The mean circulation, generally called inverse estuarine, is characterized by two counter-owing currents: in the upper layer Atlantic Water, with a salinity of about 36:2 psu, ows eastward spreading into the Mediterranean Sea and in the lower layer, Mediterranean Water with a salinity of about 38:4 psu, ows westward toward the Atlantic Ocean (Lacombe and Richez, 1982).

As initially suggested by Bryden and Stommel (1984), the mean circulation of the Strait can be described as a two-layer system hydraulically controlled at Camarinal Sill.
For a steady two-layer non rotating inviscid flow through a rectangular cross-section the hydraulic control occurs when

\[ G^2 \cdot F_1^2 + F_2^2 = 1, \text{ with } F_i^2 = \frac{u_i}{g h_i}; \]  

where \( G \) is the composite Froude number, \( F_i \); \( u_i \) and \( h_i \) represent respectively the layer Froude number, velocity and thickness of the upper (\( i = 1 \)) and lower (\( i = 2 \)) layers, and \( g^0 = \sqrt{\frac{g}{\rho}} \) is the reduced gravity (\( \sqrt{\frac{g}{\rho}} = \sqrt{\frac{g}{\rho_1} \frac{g}{\rho_2}} \) (for a rigorous derivation of (1) see Armi, 1986 and Lawrence, 1990). At the hydraulic control locations the flow undergoes a transition from subcritical (\( G^2 < 1 \)) to supercritical (\( G^2 > 1 \)). A hydraulic jump occurs instead when the flow undergoes a transition from supercritical to subcritical. Hydraulic jump is characterized by turbulent dissipation of energy.

Adding to mass and salt conservation the further constraints of a hydraulic control at Camarinal Sill and the overmixing criterion (Stommel and Farmer, 1953) to the Mediterranean Sea Bryden and Stommel (1984) found that the overmixed limit corresponds to a salinity difference between Atlantic Ocean and Mediterranean Sea of 1.7 psu with an in-flow and out-flow transports of 1.7 and 1.6 \( \times 10^6 \) m\(^3\) s\(^{-1}\) respectively. They also found that the interface depth, over Camarinal Sill, was located at mid-depth (\( h_1 = h_2 \)).

Motivated by the complex geometry of the strait, Farmer and Armi (1986) considered the case of a mean circulation hydraulically controlled both at Camarinal Sill and at the contraction near the exit of the surface flow into the Mediterranean. Applying these two hydraulic constraints in a steady two-layer non rotating inviscid flow model they found that the exchange flow rate through the strait reaches a maximal value. Differently from Bryden and Stommel they found that the minimum salinity difference admitted between the Atlantic Ocean and the Mediterranean Sea is about 2.0 psu: The maximum exchange
for the model of Farmer and Armi is also characterized by a deeper interface between the two layers over Camarinal Sill and a transport 20% lower than those predicted by Bryden and Stommel.

To explain the observed seasonal cycle of the surface inflow, deduced from the sea level difference between Gibraltar and Ceuta, Bormans et al. (1986) found that the exchange rate through the strait is mainly submaximal, i.e. the inflow through the strait is controlled only over Camarinal Sill.

In order to improve the models of Bryden and Stommel (1984) and Farmer and Armi (1986) other features, such as Earth's rotation, friction and nonrectangular cross sections for the configuration of the strait, have been added in literature. Assuming potential vorticity conservation Bormans and Garrett (1989a) estimated the influence of the Earth's rotation on the inflow. They found that the main effect of the Earth's rotation is to produce a tilt across the strait of the interface between the two layers, which is minimum at Camarinal Sill section, while is enhanced to the east of Tarifa Narrows. Also Dalziel (1990) estimated the influence of the Earth's rotation on the inflow predicting a decrease in the exchange up to one-third with respect to the non rotating case. Including friction in the model of Farmer and Armi (1986), Bormans and Garrett (1989b) estimated the effect of both interfacial and bottom friction. They concluded that, using reasonable drag coefficients (compatible with dissipation measurements by Wesson and Gregg (1988)), the interfacial friction has a smaller influence on the exchange than the bottom friction. They also found that, by including the bottom friction on the sloping sides of the strait, the maximal and submaximal solutions tend to come closer than in the inviscid model. Using more realistic bathymetry, modelled via triangular or parabolic cross-strait sections,
Bormans and Garrett (1989b) and Dalziel (1988) showed that the exchange is reduced and the interface between the two layers is raised compared with a simple rectangular cross-section bathymetry.

Using a three-dimensional general ocean circulation model, Wang (1989) was able to describe most of the aspect of the mean circulation of the Strait of Gibraltar. However, because of the relatively low horizontal and vertical resolutions (\(\approx 5 \text{ km} \) and 50 \text{ m} respectively), the model did not reproduce the hydraulic control over Camarinal Sill. Wang found in fact that while the surface flow is supercritical, i.e. hydraulically controlled on the eastern entrance the bottom flow is subcritical, i.e. not controlled on the western entrance, over Camarinal Sill. To our knowledge, Wang's model is the only three-dimensional ocean circulation model applied to the Strait of Gibraltar in the last ten years.

In the present work we describe a numerical investigation of the mean exchange of the Strait of Gibraltar using the Princeton Ocean Model (POM) (Blumberg and Mellor, 1987) in a very high-resolution configuration (\(\xi x; \xi y < 500 \text{ m} \) within the strait) in order to represent well all dominant topographic features of the strait. In particular, the model described in this paper simulates the nature of the mean exchange through the strait, i.e. whether it is maximal or submaximal by determining the locations of the hydraulic control.

The paper is organized as follows: in section 2 the model is described with particular attention to the improvements we introduced in the original POM code in order to better describe the physics of the Strait. Model results and discussion about the computation of the Froude number are described in section 3. Finally, section 4 contains the summary and conclusion.
2. Model description

The model used for this study is the three-dimensional, sigma coordinate POM model developed by Blumberg and Mellor (1983). It is a free surface - primitive equations - hydrostatic model. It has been extensively used for a wide range of oceanic problems including estuarine and shelf circulation studies (Blumberg and Mellor, 1983) (Oey et al., 1985), data assimilation in the Gulf Stream (Mellor and Ezer, 1991) and general circulation studies in the Mediterranean Sea (Zavatarelli and Mellor, 1994).

The model numerically solves, in finite difference form the momentum equation, the continuity equation and the tracer (temperature and salinity) equations along with a non-linear equation of state \( \frac{1}{\rho} = \frac{1}{\rho} \mu \left( S ; P \right) \) (Mellor, 1991b) which couples the two active tracers to the fluid velocity. Vertical turbulent mixing processes are parameterized with the 2.5 order turbulent closure sub-model of Mellor and Yamada (1982). The horizontal momentum, heat and salt small scale mixing processes are parameterized as horizontal diffusion (along sigma surfaces), depending on the horizontal velocity shear and on the grid spacing via the Smagorinsky diffusion scheme (Smagorinsky, 1963):

\[
A_H(x; y; t) = A_M(x; y; t) = C \xi x \xi y \frac{\mu}{h} N^2(x; y; t) + S^2(x; y; t) \]  

(2)

where \( \xi x \xi y \) are the grid sizes, \( C \) is a constant taken to be 0.2 in this study, and \( S \) and \( N \) are the mean shear and the normal stress \( N(x; y; t) = \frac{\partial}{\partial x} u(x; y; t) \) i \( \frac{\partial}{\partial y} v(x; y; t) \); \( S(x; y; t) = \frac{\partial}{\partial y} v(x; y; t) \) i \( \frac{\partial}{\partial x} u(x; y; t) \).

The model uses an explicit leapfrog scheme for time stepping, except for the vertical diffusion terms, which are treated with an implicit scheme. To provide free surface variations, in explicit differencing, the model also solves, with a small time step, a set of vertically integrated equations of continuity and motion, usually called external mode.
For computer time economy the 3D-equations, usually called internal mode, are solved with a larger time step, limited by the CFL condition for the internal gravity wave speed, using a time splitting technique. The external and internal time steps used in this study are of 1 sec. and 60 sec., respectively.

The model specifies the values of all variables at the nodes of a curvilinear orthogonal grid, staggered as in Arakawa-C scheme, conserving linear and quadratic quantities like mass and energy. Details on the transformed equations in \( \tilde{\phi} \)-coordinate system and the numerical algorithm can be found in (Mellor, 1991a).

2.1. The model grid and bathymetry

The region covered by our model includes the Strait of Gibraltar and the two sub-basins connected to it, i.e., the Gulf of Cadiz and the Alboran Sea. The horizontal model domain extends longitudinally from 10\(^{\circ}\)W to 4\(^{\circ}\)E and meridionally from 33\(^{\circ}\)N to 39\(^{\circ}\)N and is covered with a coast-following curvilinear orthogonal grid made by 306 \( \times \) 53 grid points (Figure 2). Note that the resolution is much higher in the strait (\( \approx \) 500 m) than in the eastern (8; 15 K m) and western ends (10; 20 K m); so that the dynamics in the strait will be well resolved. The vertical model grid is made by 25 sigma levels. The \( \tilde{\phi} \)-coordinate is defined as \( \tilde{\phi} = \left( z - \tilde{z} \right) = \left( H + \tilde{z} \right) \); it ranges from \( \tilde{\phi} = 0 \) at \( z = \tilde{z} \) (at the surface) to \( \tilde{\phi} = 1 \) at \( z = H \) (at the bottom). The sigma levels follow a logarithmic distribution at the surface, in order to achieve a better resolution of the upper layer stratification, and a uniform distribution in the rest of the water column. The vertical distribution of the \( \tilde{\phi} \) levels for different water depths is shown in table 1.

The model topography was constructed by bilinear interpolation of the depth data onto each grid point of the horizontal model grid. The depth data were obtained by merging
the high resolution (<1 km) topographic data set of the Strait of Gibraltar, provided by the LODYC laboratory, with the relatively low resolution (5 min) U.S. Navy DBDB5 data set (1983) for the Alboran Sea and the Gulf of Cadiz.

In an attempt to reduce the well-know pressure gradient error produced by sigma coordinate grids in regions of steep topography (Haney, 1991) an additional smoothing was applied where the slope ($\pm H = H$) was greater than 0.2, as suggested by Mellor et al., (1994). In order to estimate the residual pressure gradient error, the model was integrated for one year without initial horizontal density gradient, i.e., with salinity and temperature fields varying only with depth, with no open boundary applied, i.e. closed domain, and without any other external forcing. The maximum intensity of erroneous currents introduced by the sigma coordinates was of 2 cm s$^{-1}$: Since the expected baroclinic velocities are up to 1 m s$^{-1}$ this error seems to be tolerable.

The resulting model topography, with the minimum depth of the shelf set to 25 m, is illustrated in Figure 3; in particular, the bottom topography and the computational grid in the region of the strait is shown in Figure 4. Here are clearly recognizable the dominant topographic features of the strait (from west to east): Spartel Sill (Sp), Tangier basin, Camarinal Sill (Cm) with a depth of 284 m and Tarifa Narrows.

2.2. Boundary and initial Conditions

In the vicinity of the eastern and western ends of the computational domain two open boundaries are defined, on which values of velocity, temperature and salinity must be specified. In order to minimize the contamination of the interior model solution due to wave reflection at the boundaries, an Orlanski radiation conditions (Orlanski, 1976) was used for the depth-dependent velocity, and a flow relaxation scheme (FRS hereinafter)
(Martinsen et al., 1987) was used for the depth-integrated velocity. The aim of the FRS scheme is to relax the prognostic barotropic velocity toward a null transport within specified regions, generally called FRS zones, near the open boundaries. If $U_i$ is the prognostic barotropic variable computed in the interior of the model domain, the relaxed variable in the FRS zones is given by:

$$U = \bar{U}_0 + (1 - \bar{\alpha}) U_i; \quad (3)$$

$U_0$ represent the null transport and $\bar{\alpha}$ is the relaxation parameter, varying from 1 at the open boundary to 0 at the inner edge of the FRS zones. The following polynomial representation, proposed by Jensen (1998), was used for $\bar{\alpha}$

$$\bar{\alpha}(I) = [(1 - q) I(I) + q]^p , \quad I = 1; 2; \ldots; 10 , \quad (4)$$

with $p = 6$ and $q = 0$. Here $I(I)$ is the distance of the $I$-th grid element from the open boundary and 10 is the number of grid elements in the relaxation zones used in the model. A Newtonian restoring was used for the two active tracers, with a damping timescale of 5 days. Temperature and salinity are restored at all vertical levels towards the initial condition, in the same FRS zones defined for the barotropic velocity.

The normal velocities are set to zero along coast boundaries. At the bottom, adiabatic boundary conditions are applied to temperature and salinity while a quadratic bottom friction with a prescribed drag coefficient is applied to the momentum flux. This is calculated by combining the velocity profile with the logarithmic law of the wall:

$$C_D = \max 2.5 \times 10^3; \frac{k^2 \ln (\frac{z_b}{z_0})}{k^2 \ln (\frac{z_b}{z_0})} ; \quad (5)$$
where $k$ is the Von Karman constant, $z_0$ is the roughness length, set to 1 cm, and $\zeta \cdot z_b$ is the distance of the deepest velocity grid point from the bottom. Wind and tidal effects, as well as surface fluxes, are not included in our simulation.

The model is started from rest; it is only driven by an initial density difference without any other forcing. To do this we have filled the model with two water masses, horizontally uniform and vertically stratified, separated by an imaginary dam in the middle of the strait (Lon: $5^\circ42^\prime W$) that is removed at the initial time. Using this procedure, the transport of the two layers in the strait is able to freely adjust to the density gradient as it does in nature. This kind of initialization is generally known as lock-exchange initial condition and it has already been used to study the general circulation of the western Mediterranean Sea (Herbaut et al., 1996).

The centered difference scheme used in POM to solve the tracer transport is not able to simulate the free flow adjustment through the strait for a lock-exchange initial condition. The standard scheme is conservative but dispersive, creating spurious temperature and salinity in regions with sharp density gradients (Gross et al., 1999). Thus, in order to reduce the large numerical error, the Smolarkiewicz upstream-corrected scheme (Smolarkiewicz 1984, 1990) was implemented and used in the model, instead of the standard centered advection scheme. The Smolarkiewicz scheme, which is a flux corrected upstream scheme characterized by a small implicit diffusion, has been already used in other oceanographic models giving satisfactory results: see for example Wang (1984, 1989), Speich (1992) and Herbaut (1996). The scheme was implemented as an iterative algorithm: in the first iteration a classical upstream scheme is used, while each following "corrective" iteration reapplies the upstream scheme using a specially defined anti-diffusive velocity.
The number of iterations is optional; each additional iteration increases the solution accuracy as well as the computing time: the number of iterations chosen for this study is three, which gives sufficient accuracy for our purposes.

Initial temperature and salinity fields for the Alboran basin have been obtained by a horizontal average, over the Alboran Sea, of the spring MODB data. Initial fields for the western basin have been obtained horizontally averaging, over the Gulf of Cadiz, the temperature and salinity of the spring Levitus data (Levitus, 1982). In Figure 5 are shown the initial temperature and salinity profiles for the Alboran basin and the Gulf of Cadiz, respectively.

3. Model results.
3.1. Spin-up phase

Immediately after the "dam" is lifted, a sharp cross-strait density discontinuity of 1:4 \( \frac{3}{4} \) is created between the heavier Mediterranean water and the lighter Atlantic water. The generated along-strait pressure gradient drives the initial motion of the two water masses: Atlantic and Mediterranean waters spread outward, in opposite directions and in geostrophic balance, over the whole dam width, in agreement with the Wang's model (1985).

The first internal Rossby radius \( (R) \) within the strait, in our model, is about 14 K m. It was calculated using \( R = \frac{p}{g^0} \) (LeBlond and Mysak, 1978), where \( g^0 \) is the reduced gravity, \( f \) is the Coriolis parameter corresponding to 36\(^{\circ}\)N and \( h_e \) is the equivalent depth defined as \( h_e = h_1 h_2 = (h_1 + h_2) \); where \( h_1 \) and \( h_2 \) are the thicknesses of the surface Atlantic inflow and of the deep Mediterranean outflow respectively. Thus, since the width of the strait is of the same order of the first internal Rossby radius, the two flows
are quickly trapped by the coast. At the coast there is no upstream component of the Coriolis acceleration. As a consequence, two superimposed density currents begin to form and propagate, with a frontal speed of 57 cm s\(^{-1}\) and 69 cm s\(^{-1}\) for the Atlantic and Mediterranean waters, respectively (Figure 6). The lower Mediterranean gravity current reaches the western strait opening after 6 hours, while the surface Atlantic gravity current reaches the eastern strait after 15 hours.

3.2. Mean flow

In the first three days of integration the abrupt along-strait vertically integrated density difference (\(\frac{\rho}{\rho_A} = 1.4 \frac{10^3}{\mu}\)), between the extreme edges of the strait, generates a baroclinic transport of about 0.8 Sv in both layers. In the following six months the transport decreases as the along-strait density difference becomes smoother (Figure 7). After 200 days of integration the along-strait density difference reaches a quasi-steady values of 0.99 \(\frac{\rho}{\rho_A}\) that generates a quasi-steady mean transport in both layers and an outflow salinity transport of 0.72 Sv and 1.39 \(10^3\) m\(^3\) s\(^{-1}\); respectively (Figure 8). The inflow (\(Q_A\)) and outflow (\(Q_M\)) transports are computed integrating the along-strait velocity vertically, from the bottom up to the surface, and then laterally across section C (refer to model grid in Figure 4):

\[
Q_M(t) = \int_C ds \int_{\frac{\rho}{\rho_A} = 1}^{\frac{\rho}{\rho_A} = 0} u_{out}(z,t) \, dz
\]

\[
Q_A(t) = \int_C ds \int_{\frac{\rho}{\rho_A} = 1}^{\frac{\rho}{\rho_A} = 0} u_{in}(z,t) \, dz;
\]

while the outflow salinity transport (OST) is computed by an analogous integration of the along-strait outflow velocity times the salinity excess above a mean salinity of Atlantic
inflow of 36:1 psu:

\[
\text{OST}(t) = \frac{Z}{c} \int_{S=0}^{S=36:1 \text{ psu}} u_{\text{in}}(z;t)(S(z;t)-36:1 \text{ psu}) \, dz:
\]

(8)

The values of the simulated transports are in good agreement with recent estimates on the direct measurements carried out by Bryden et al. (1994) and Tsimplis and Bryden (2000).

The model also reproduces the two-layer character of the inflow within the strait, as shown in the scatter plot of along-strait steady velocity versus salinity in Figure 9a. Here there is a strong inflow of Atlantic water with salinities less than 36:5 psu and a strong outflow of Mediterranean water with salinities greater than 38:0 psu. For salinities ranging between 37:0 psu and 37:5 psu there are both inflow and outflow. In Figure 9b the along-strait mean velocity profile versus salinity is shown, obtained by averaging the velocities over salinity layers of thickness \( \delta S = 0.2 \) psu. For salinities less than 37:25 psu there is an inflow velocity that reaches a maximum value of 40 cm s\(^{-1}\) at a salinity of 36:25 psu. For salinities greater than 37:25 psu there is an outflow velocity that reaches a maximum value of 35 cm s\(^{-1}\) at a salinity of 38:4 psu. The transition between inflow and outflow then occurs at a salinity of 37:25 psu, which will be hereinafter taken to define the interfacial isohaline between the Atlantic and Mediterranean waters. Such definition is in agreement with the choice by Lacome and Richez (1982), Bormans and Garrett (1989b), and Bryden et al. (1994).

In order to quantitatively compare the model results with observed data we have made a linear regression (Figure 10) comparing predicted and observed mean velocity profiles for eight current meter moorings of the Gibraltar Experiment, a large-scale field study carried out for one year period between October 1985 and October 1986 (Figure 1).
mean velocity data are taken from Candela et al. (1990). The mean errors obtained for the upper and lower layer are respectively 2:1 and 3:8 cm s$^{-1}$.

Figure 11 shows the salinity structure for the cross-strait sections near Spartel sill (A), Camarinal Sill (B) and Tarifa Narrows (C and D) respectively, after 360 days of integration (hereinafter the model's outputs will be always referred to 360 days of integration). It is evident that in all cross-sections that the light Atlantic water tends to pile up against the south shore, increasing the thickness of the eastward upper-layer flow toward south, while the heavy Mediterranean water tends to pile up against the north shore, increasing the thickness of the westward lower-layer flow toward north. At section A the depth of the 37:25 psu isohaline varies from 165 m on the southern side to 122 m on the northern side, with a cross-strait slope of 1:7 $\times$ 10$^{-3}$. Near Camarinal Sill the cross-section slope of the interface increases up to 4:7 $\times$ 10$^{-3}$ while along Tarifa Narrows the cross-strait slope of the interface decreases down to 2:8 $\times$ 10$^{-3}$. The predicted value of the cross-strait slope near Camarinal Sill is in agreement with the observed value of 4:6 $\times$ 10$^{-3}$ given by Bryden et al. (1994). In contrast with Bormans and Garrett (1989a) the rotational effect seems to be smaller at the Tarifa Narrows and Spartel Sill sections than at the Camarinal Sill section.

Figure 12 shows the predicted along-strait velocity distribution along the cross-strait sections A, B, C and D. Cross-strait shear of the along-strait velocity is evident at Tarifa Narrows where a surface jet is located on the north side of the strait. Such cross strait distribution of the surface velocity is in agreement with the hypothesis of constant Potential Vorticity (PV) formulated by Bormans and Garrett (1989a). In fact, as shown in Figure 13; the isolines of constant PV for the upper layer coincide, in the northern
side of strait, with the streamlines. Thus, the upper layer flow, moving eastward, has to compensate the reduction of thickness by decreasing the absolute vorticity \( f \frac{i}{\partial u/\partial y} \) and therefore increasing the northward gradient velocity \( \frac{\partial u}{\partial y} \). Nevertheless it is interesting to note that the region where trajectories and isolines of PV coincide starts just to the west of Camarinal Sill and not from an Atlantic reservoir located to the west of Spartel sill as suggested by Bormans and Garrett (1989a).

The Mediterranean outflow is westward everywhere with velocity gradually increasing from Tarifa Narrows to Camarinal Sill. A sketch of the simulated Mediterranean outflow is shown in Figure 14: the region of the strait delimited by thick lines corresponds to denser fluid with a westward speed greater than 20 cm s\(^{-1}\); whereas lines with arrows indicate the paths of jet. This picture illustrates the formation of a westward jet, at the slope beginning east of Camarinal Sill. When the jet arrives in the middle of sill, it splits in two parts, with the strong branch in the north. Subsequently the flow reaches the Tangier basin and the northern branch of the jet vanishes, whereas the other part intensifies to over 100 cm s\(^{-1}\) creating a deep jet that intrudes in the Gulf of Cadiz.

The Mediterranean flux \( h^2u^2\frac{\partial y}{\partial y} \) through the three cross-sections \( B_1; B_2; B_3 \); located between Tarifa Narrows and Camarinal Sill (refer to the strait map in Figure 14), shows maximal value in the middle of channel; while, the along strait velocity shows the growing of a jet to the east of Camarinal Sill, near the southern side (Figure 15).

In our simulation the PV for the Mediterranean layer is conserved only in regions where bathymetric slope is moderate. For this reason we believe that the formation of the deep westward jet must be due to other causes. Thus, to better analyze the mechanism of lower layer jet formation between Tarifa and Camarinal Sill, we examined the interface depth,
pressure and density at 200 m depth, along the longitudinal sections E₁; E₂ and E₃ (Figure 16). We have previously observed that the interface slope increases moving from Tarifa to Camarinal; in particular Figure 16a shows that the interface depth, along the southern sections (E₃; E₂), rises non linearly from cross section B₃ to B₁; and Figure 16b indicates a sensible density decrease along the southern section E₃. Both these features produce a decrease of pressure (Figure 16c), more evident along the southern section (E₃) than in the central part. Therefore a northward positive pressure gradient is generated, with a maximum just at Camarinal Sill. Consequently, the along strait velocity must increase in the westward direction, to balance, at least partially, the northward pressure gradient along the south side, causing the jet formation. In Figure 17a we plot the along strait lower layer velocity at 200 m depth for three cross-sections: one at Camarinal Sill and the others shifted by three grid points to the west and the east of the sill, respectively. It is evident that to the east of the sill the jet is near the south side whereas to the west of the sill it moves to the north side. This behavior is related to the vertical mixing just west of the sill due to the abrupt acceleration of the dense Mediterranean water going down the sill. Here, in fact, vertical transport changes sign in the middle of the Mediterranean layer, with positive values in the upper part and negative values in the lower part (Figure 18b). From Figure 17b it is possible to note that in the east section the salinity is quite uniform, ranging from 38.2 psu to 38.4 psu, whereas a few kilometers westward it is lower than 37.8 psu except in the north where it remains unchanged at 38.3 psu. Thus, salinity mixing produces a dramatic cross-section density gradient, inducing a maximum southward pressure gradient in the north (Figure 17c). In summary, the
maximum pressure gradient shifts toward north as soon as Mediterranean water flows over Camarinal Sill, inducing a northward shift of the jet.

In the Tangier basin the salinity is more homogeneous along cross strait sections but presents maximal values in the deeper central part of the strait, constraining the jet in this zone.

3.3. Rotational effects and stationary flow

Some authors have speculated that the effect of Earth's rotation can be evaluated assuming a geostrophic balance for the cross-strait momentum equation. In order to better understand the rotational effect within the strait, we have evaluated ten days averages of all the terms present in the momentum equation. In Figure 18 are shown the vertical profiles of pressure gradient, horizontal advection, vertical transport, Coriolis term and vertical diffusion in four points within the strait along section E. For the point just west of Camarinal Sill the geostrophic balance is present only for the upper layer, whereas the advection terms are not negligible for the Mediterranean layer. Always in this point, friction is about 30% of the maximum forcing only close to the bottom, whereas vertical diffusion contributes to the balance only in the interface layer. Mixing is particularly active in the interface also in the northern part of Tarifa Narrows. For the points near the western entrance of the strait, just east of Tarifa and near Gibraltar there is geostrophic balance in part of the lower layer but not in the upper one where the advection terms are not negligible.

The ten days mean tendency of the cross-strait velocity component vanishes everywhere, but looking at its root mean square it is possible to note that only in regions where geostrophic balance holds is there a perfect stationary flow. Where the advection terms
are important, the mean quadratic tendency of the cross-strait velocity is not vanishing and has the same magnitude of the forcing terms at the interface. This indicates that in such regions the flow is subjected to short period oscillations that are canceled when applying a time mean operator of few days.

3.4. Hydraulic Control.

To allow a comparison between the model results and the two layer hydraulic theory, it is necessary to define an interface to discriminate the Mediterranean and the Atlantic layers. As mentioned in section 3.2., we choose the 37:25 psu isohaline to define such an interface. Figure 19 shows the predicted steady interface depth within the Strait of Gibraltar. Note that the interface depth increases toward east and south with an abrupt change near Camarinal Sill.

In order to compute the layer Froude numbers ($F_i$) the interface depth is taken as the height of the upper layer $h_1$, and the lower layer thickness $h_2$ is calculated as $H - h_1$, where $H$ is the water depth. Layer velocities and densities ($u_i; \rho_i$) are computed by averaging the along-strait velocity and density from the surface to the interface depth for the upper layer, and from the interface depth to the bottom for the lower layer. The resulting value of $G^2$ is shown in Figure 20. Note that within the Strait $G^2 < 1$ everywhere, i.e. the flow is subcritical. However, this result seems to be inconsistent with the simulated character of the flow; in fact, the predicted salinity distribution along the longitudinal axis of the Strait (sec. E), shown in Figure 21; suggests the presence of a hydraulic jump west of Camarinal Sill.

It seems that, as also argued by Winters and Seim (2000), the direct application of the two-layer hydraulic theory to the Strait of Gibraltar is not obvious. As observed
by Bray et al. (1995), entrainment and mixing between the Atlantic upper layer and the Mediterranean lower layer are responsible for the creation of a third interfacial layer. Thus, in order to verify the presence of an interface layer, a quantitative method similar to that described by Bray et al. was used. In particular Bray et al. fitted the salinity profiles with three different straight lines for the upper ($t$), interface ($i$), and lower layers ($b$) respectively. Intersection of the upper layer line with interface line represents the depth of the lower boundary of the upper layer ($H_u$) while intersection of the bottom layer line with interface line represents the depth of the upper boundary of the bottom layer ($H_b$). Consequently the thickness of the interface layer is defined as the difference $H_b - H_u$. However, vertical profiles of salinity often are not well represented by these simple functions because in many points (particularly in the surface layer) salinity does not increase monotonically with depth. To overcome this problem we have fitted the salinity profiles with a cubic polynomial, conditioning the parameter of the polynomial to guarantee a monotonic increasing of salinity with depth. In Figure 22 are shown the salinity profile, cubic polynomial and interface layer for a point in the strait just east of Camarinal Sill. In Figure 22 are also shown, for the same point, velocity and density profiles. Predicted thickness of the interface layer for the whole strait is shown in Figure 23. It is interesting to note that the thickness has the greatest values just west Camarinal Sill, indicating the presence of mixing in that region. The thickness of the interfacial layer obtained is good in agreement with that calculated by Bray et al. (1995). Moreover, so in agreement with Bray et al., our simulation shows a difference in horizontal transport along the strait (Figure 24) due to vertical exchange between Atlantic and Mediterranean layers.
It is interesting to note that the largest changes occur in the region west of Camarinal Sill (grid point 104) where mixing effects are particularly strong.

To evaluate the effect of more than two layers it is necessary to extend the usual hydraulic control theory to a multi-layer flow (see Appendix A). In particular, in presence of a mixed layer we used the multi-layer formulation for three layers with Froude numbers defined as:

\[ F_1^2 = \frac{\bar{u}_1^2}{h_1 g(1 - r_{1,2})}; \quad F_2^2 = \frac{\bar{u}_2^2(1 - r_{1,3})}{h_2 g(1 - r_{1,2})(1 - r_{2,3})}; \quad F_3^2 = \frac{\bar{u}_3^2}{h_3 g(1 - r_{2,3})}; \]  

(9)

where \( r_{i,j} \) is the density ratio \( \bar{\rho}_i/\bar{\rho}_j \): The composite Froude number is given, to a good approximation, by:

\[ G^2 = F_1^2 + F_2^2 + F_3^2; \]  

(10)

The predicted composite Froude number within the strait is shown in Figure 25. Critical values of \( G^2 \) can be observed over the whole cross-strait section west of Camarinal Sill and in the northeastern part of Tarifa Narrows. The contribution from the interfacial Froude number \( (F_2^2) \) is negligible everywhere, whereas the upper value \( (F_1^2) \) is essential only for the Tarifa narrow region.

To evaluate the sensitivity of the composite Froude number to the position of the upper and lower interfaces of the mixed layer, we varied (as in Pratt et al., 1999) the position of the interfaces of \( \S \) 10 m about the predicted values, in particular we considered an error of about 25% (\( \S \) 20 m) in determining the thickness of the mixing layer (Figure 26). From the results it is evident that, in spite of the introduced errors, the composite Froude numbers are qualitatively similar in the sense that there is a supercritical regime at Camarinal Sill and a subcritical regime at Tarifa Narrows along the section E.
Thus, model results shown in Figure 25 seem to indicate the presence of hydraulic control at Camarinal Sill, whereas there is not a clear indication for Tarifa Narrows. The uncertainties come from the application of the two-dimensional hydraulic control theory to a three-dimensional flow. In an attempt to eliminate these uncertainties, we calculated the cross-strait integrated expression of the Bernoulli Potential $B_e$. For example, in the two layer formulation the first layer integrated potential is:

$$Z \int_{y_n}^{y_s} B_{el}(x; y) \, dy = Z \int_{y_n}^{y_s} \frac{\nabla h_1(x; y)}{2} \, dy + g \int_{y_n}^{y_s} \frac{\nabla h_1(x; y)}{2} \, dy + \int_{y_n}^{y_s} \left( \frac{\nabla h_1(x; y)}{2} \right) \left( h_5(x; y) + h_2(x; y) \right) \, dy;$$

(11)

where $h_5$ is the bottom depth, $y_n$ and $y_s$ the north and south boundary respectively (note that the overbar indicates vertical average). To apply hydraulic theory is necessary to express the Bernoulli Potential and the volume flow rate $Q$ in terms of cross-strait integrated quantity:

$$Q_1 = \int_{y_n}^{y_s} h_1(x) \, u_1(x) \, dy = C_{q1} f h_1(x) g f h_1(x) g$$

(12)

with:

$$C_{ul} = \frac{f h_1^2 g}{f h_1 g f h_1 g}, \quad C_{rh11} = \frac{f h_2 g}{f h_2 g f h_1 g}, \quad C_{rh12} = \frac{f h_2 g}{f h_2 g f h_2 g},$$

$$C_{q1} = \frac{f h_1^2 h_1 g}{f h_1 g f h_1 g}, \quad C_{rh1S} = \frac{f h_5 g}{f h_5 g f h_5 g};$$

(14)

Equations and coefficients for the second layer can be easily deduced. Coefficients $C_{ul}$, $C_{rh11}$, $C_{rh12}$, $C_{q1}$, $C_{rh1S}$ (and the same for the second layer) are not constant along the strait, but if we define two smoothed functions for each coefficient, representing higher
and lower values at every along strait position, we can neglect their \( x \)-derivative. Hence, differentiating in the along-strait direction the expressions (13; 14) for the higher and lower set of coefficients, and dividing for the width of the strait \( b(x) \), we obtain the same formulation of Froude numbers in terms of mean cross-strait values of velocity current and layer thickness. For two layer we obtain:

\[
\begin{align*}
F_1^2 &= \frac{C_{u1} f \bar{b} g}{C_{rh11} g h_1}; \\
F_2^2 &= \frac{C_{u2} f \bar{b} g^2}{C_{rh22} g h_2}; \\
g &= g 1 i \frac{\tilde{A}}{\frac{1}{2} C_{rh22}} \\
\end{align*}
\]

indicating higher and lower values of coefficients as \( \max hC_i \) and \( \min hC_i \); we can limit the values of Froude numbers between:

\[
\begin{align*}
\max D F_i^2 E &= \frac{\max hC_{u1} i f \bar{b} g^2}{\min hC_{rh11} i \min hg i h_i}; \\
\min D F_i^2 E &= \frac{\min hC_{u1} i f \bar{b} g^2}{\max hC_{rh11} i \max hg i h_i}; \\
g &= g 1 i \frac{\tilde{A}}{\frac{1}{2} \max hC_{rh22} i} \\
\end{align*}
\]

An analogue formulation can be found for three layers and the resulting maximum and minimum composite Froude numbers are plotted in Figure 27. It is interesting to note that significant difference between the two curves is present only at Camarinal Sill and at the beginning of Tarifa Narrows. However the lower curve overtakes critical values only at Camarinal Sill, whereas at Tarifa Narrows not even the upper limit is greater than 0.8.

4. Summary and conclusions

The simulation of the mean exchange through the Strait of Gibraltar is a fascinating challenge for ocean modelers. Previously, this problem was examined by Wang (1989) with a three-dimensional z-level model having a horizontal domain limited only to the strait region, with horizontal and vertical resolution of about 5 km and 50 m respectively. Wang was able to describe most of the aspects of the mean circulation, but the model failed to reproduce the hydraulic conditions within the strait.
Coast-following curvilinear grid and terrain-following vertical grid represent indispensable features that greatly improve the ability to reproduce the strait circulation. For these reasons, we choose the three-dimensional sigma coordinate Princeton Ocean Model to study the mean exchange.

Since the transport of salinity and buoyancy is the principal mechanism involved in the steady strait circulation, it is crucial to use a conservative, non dispersive and non dispersive numerical scheme for the advection of horizontal tracers. Therefore, we have implemented and used the Smolarkiewicz (1984) scheme, instead of using the standard POM’s scheme.

The real dynamics of the Strait of Gibraltar is influenced by non stationary barotropic forcing as the astronomical tide (Helfric, 1995) and by the evolution of the mean atmospheric pressure on the Atlantic Ocean and the Mediterranean basin (Harzallah et al., 1993). However, the first purpose of this study was to demonstrate the ability of our model to reproduce the mean exchange and the hydraulic characteristics of the strait circulation. The results obtained for integral quantities like the mean transport ($0.72 \, \text{Sv}$) and the mean outflow salinity transport ($1.39 \times 10^3 \, \text{m}^3\,\text{s}^{-1}$), as well as the depth and shape of interface are in agreement with the actual estimates deduced from measurements and reported in literature. Moreover, the simulated mean currents are in agreement with the available measurements obtained from the large-scale Gibraltar Experiment. The circulation described by our numerical model is very detailed, showing previously unknown features of the mean strait circulation, but the lack of observations cannot help us to confirm or refute them. In particular, the results for the Mediterranean layer indicate the formation of a jet that is located east of Camarinal Sill, near the south side, whereas
westward of the sill the flow is subject to a hydraulic jump and the jet is flattened and attached on the north border.

The second purpose of this study was to evaluate, in the frame of hydraulic theory, if there are regions of hydraulic control and consequently if the Strait of Gibraltar is in a submaximal or maximal regime. It has been stressed that, because of the presence of a mixing layer, three layers structure gives a better representation of the circulation within the strait. For this reason we used a formulation of Froude numbers for three layers indicating the presence of hydraulic control over the whole cross-strait section west of Camarinal Sill and in the northern part of Tarifa Narrows. Model results seem to indicate the presence of hydraulic control at Camarinal Sill, whereas there is not a clear indication for Tarifa Narrows. To remove this uncertainty we formulated cross-strait mean equations in terms of cross-strait mean quantity and confined the solution between two upper and lower limit solutions. This procedure allowed us to give upper and lower limits for the values of the composite Froude number for the cross-strait mean flow. In particular, at Camarinal Sill the mean cross-strait composite Froude number is found in the range 1:2 to 1:5; whereas at Tarifa Narrows between 0:4 to 0:8. This result allows us to conclude that the mean circulation simulated by the model is in a sub-maximal regime.

Finally, in spite of all the limitations relative to the neglect of non stationary forcing this model represents the first necessary step toward a complete model analysis of the Gibraltar Strait circulation that will include, in a future work, the astronomical tide, atmospheric pressure on adjacent basins and the real seasonal cycles of salinity and temperature.
Appendix A: Hydraulic Control for multi layer flow

The motion of a n-layer flow is governed by continuity and Bernoulli equation applied to each layer (Baines, 1988):

\[ h_k \frac{\partial u_k}{\partial x} + u_k \frac{\partial h_k}{\partial x} = Q_k \frac{1}{b_k(x)} \quad k = 1; 2; 3; \ldots; n \quad (A1) \]

\[ u_k \frac{\partial u_k}{\partial x} + \frac{g}{\gamma_k} \sum_{j=1}^{k} \frac{1}{\gamma_j} \frac{\partial h_j}{\partial x} + \sum_{j=k+1}^{n} \frac{g}{\gamma_j} \frac{\partial h_j}{\partial x} = i g \frac{\partial b_k}{\partial x} \quad k = 1; 2; 3; \ldots; n \quad (A2) \]

where \( u_k; h_k; Q_k; \gamma_k \) and \( b_k \) are respectively the velocity, thickness, volumetric flow rate, density and width of the \( k \)th layer. In analogy with the treatment of Armi (1986) for a two-layer flow, the above quasi-linear system of differential equations, can be written in the general form:

\[ \mathbf{C} \mathbf{v} = \mathbf{f} \quad (A3) \]

where \( \mathbf{v} \) and \( \mathbf{C} \) are defined as:

\[ v_j = \left( \frac{\partial u_j}{\partial x} \right)_{n+1; j; 2n}; \quad \mathbf{C} = \left[ \begin{array}{cc} U & G \\ H & U \end{array} \right] \quad (A4) \]

In particular \( U; H \) and \( G \) are defined as:

\[
U = \begin{bmatrix}
2 & u_1 & 0 & \cdots & 0 \\
6 & 0 & u_2 & \cdots & 0 \\
4 & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & u_n
\end{bmatrix}
\]

\[
H = \begin{bmatrix}
2 & h_1 & 0 & \cdots & 0 \\
6 & 0 & h_2 & \cdots & 0 \\
4 & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & h_n
\end{bmatrix}
\]

and

\[
G = \begin{bmatrix}
2 & g & g & \cdots & g \\
6 & g h_2 & g & \cdots & g \\
4 & \vdots & \ddots & \ddots & \vdots \\
0 & 0 & 0 & \cdots & g
\end{bmatrix}
\]

with \( g_i = g^{1/2} \).

When the flow is hydraulically controlled the \( \det(\mathbf{C}) = 0 \) (Armi, 1986). A simple way to calculate \( \det(\mathbf{C}) \) is to gather terms containing the product of all thickness and permuting
the kth and \((n + k)\)th rows of C for terms containing the product of \(n\) 1 thickness:

\[
\begin{align*}
\text{jC}_j &= (i \ 1)^n f_{h_1} h_2 \cdots h_n \text{jG}_j i \ u_1 h_2 \cdots h_n \ \cdots \ h_1 u_2 h_3 \cdots h_n \\
&\quad g_{ln} \ g_{ln} \ g_{ln} \ g_{ln} \ g_{ln} \ g_{ln}
\end{align*}
\]

where \(\mathbb{L}\) contains terms combining two and more layer velocities \(u_k\). Previous formulation can also be written in a more compact form:

\[
\text{jC}_j = (i \ 1)^n h_k g_i jM_{hj} \ i \ u_k g_i h_j \ A \ @ \ h_j \ A \ jM_{ukj} + \mathbb{L}; \quad (A7)
\]

with \(M_h(ij) = S(ij) = \frac{1}{g} \ i < j \quad \text{and} \quad M_{uk}(ij) = \begin{cases} u_k & i = k \ j = k \\ 0 & i = k \ j \neq k \end{cases} \quad (A8)

Remembering that the determinant of a matrix \(S\) defined as:

\[
S = \begin{vmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix}
\]

\[
\text{det}(S) = \text{det}(W) \quad (A9)
\]

can be calculated as \(\text{det}(S) = \text{det}(W)\) where \(W(ij) = S(ij) = 1\), so both \(\text{det}(M_h)\) and \(\text{det}(M_{uk})\) can be easily calculated:

\[
\text{jM}_{hj} = \begin{vmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & 1 & \cdots & 1 \\ r_{1n} & r_{2n} & r_{3n} & \cdots & r_{kn} \end{vmatrix} = \begin{vmatrix} r_{12} & i & 1 & 0 & \cdots & 0 \\ r_{13} & i & 1 & r_{23} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ r_{1n} & i & 1 & r_{2n} & i & 1 & \cdots & r_{kn} & i & 1 \end{vmatrix} = (l \ i \ r_{12})(1 \ i \ r_{23})(1 \ i \ r_{34})(1 \ i \ r_{45}) \cdots (1 \ i \ r_{n1 \ n}) \quad (A10)
\]
The same expression of Froude number can be found starting from the control condition where \( F_{k} = \frac{u_{k}^{2}}{h_{k}g(1_{i} r_{k1}) (1_{i} r_{kk+1})} \) (A14).

If we neglect the contribution of \( \mathcal{F} \) we get an expression like the two layer formulation where \( F_{k} \) is the Froude Number of the \( k \)-th layer and \( G \) is the composite Froude number.

The same expression of Froude number can be found starting from the control condition indicated by Engqvist (1996) and Baines (1988):

\[
C_{\text{HD}}^3 = \mathcal{F} \quad \text{and} \quad \det(C) = 0; \quad \text{where} \quad (A15)
\]

\[
C = \begin{pmatrix}
\vdots & \vdots & \vdots \\
\ldots & \ldots & \ldots \\
\ldots & \ldots & \ldots \\
\end{pmatrix}
\]

\[
\text{is the composite Froude number.}
\]
and \( F_i^2 = \frac{u_i^2}{g} H_1 \); \( \varphi = \left( \frac{1}{2} \varphi_1 \right) \). \( h_{i;x} = \vec{a}_{i} = \vec{c} \); In this case \( h_i \) represents the base height of the \( i \)th layer. Rendering explicit the system of equations (A14) we have:

\[
\begin{align*}
[1] & \quad C_{11} h_{1;x} + C_{12} h_{1;x} = 0 \\
[2] & \quad C_{21} h_{1;x} + C_{22} h_{1;x} = 0 \\
[3] & \quad C_{31} h_{1;x} + C_{32} h_{1;x} + C_{33} h_{1;x} + C_{34} h_{1;x} = 0 \\
[4] & \quad C_{43} h_{1;x} + C_{44} h_{1;x} + C_{45} h_{1;x} = 0 \\
\vdots \\
[n] & \quad C_{n1} h_{1;x} + C_{n2} h_{1;x} = 0 .
\end{align*}
\]

\( \sum_{n=1}^{n} C_{nn} \) (A17)

It is possible to reduce the system (A16) of \( n \) equations to \( n - 1 \) eliminating \( h_0 \) in the second equation:

\[
\begin{align*}
[2] & \quad C_{22} h_{1;x} + C_{23} h_{1;x} = 0; \\
\text{where } & \quad C_{22} = \begin{pmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{pmatrix}, \quad \text{and } C_{23} = C_{23} C_{11}; \\
\text{(A18)}
\end{align*}
\]

If \( C_{22} = 0 \) there is a control condition that correspond to the second layer stagnant (see Engqvist (1996)). Replacing the \( C_{ij} \) terms we yield:

\[
C_{22} = \frac{a}{2} \left( \frac{1}{2} + \frac{a}{2} \right) \frac{1}{2} \frac{1}{2} F^2_1; \quad \frac{a}{2} \left( \frac{1}{2} + \frac{a}{2} \right) \frac{1}{2} \frac{1}{2} F^2_2 = 0; \quad \text{(A20)}
\]

that is:

\[
\begin{align*}
\frac{3}{g} \frac{u_1^2}{\frac{a}{2}} + \frac{3}{g} \frac{u_2^2}{\frac{a}{2}} + \frac{3}{g} \frac{u_3^2}{\frac{a}{2}} \frac{u_2^2}{\frac{a}{2}} = 1; \quad \text{(A21)}
\end{align*}
\]

As the second layer is stagnant the control condition is only on the first layer:

\[
F_1^2 = \frac{3}{g} \frac{u_1^2}{\frac{a}{2}} \frac{u_1^2}{\frac{a}{2}} = 1; \quad \text{(A22)}
\]

If \( C_{22} \neq 0 \) we get a \( n \) \( 1 \) system of linear equations composed by the modified second relation equation \( C_{22} h_{1;x} + C_{23} h_{1;x} = 0 \) and the others \( 3 \) \( n \) unmodified equations. If we eliminate also \( h_{1;x} \) from 33 we get the modified relation:

\[
[3] C_{33} h_{2;x} + C_{34} h_{1;x} = 0; \quad \text{(A23)}
\]

valid only if \( C_{33} = \begin{pmatrix} C_{32} & C_{33} \\ C_{32} & C_{33} \end{pmatrix} \neq 0; \quad \text{(A24)}
\]

\( \text{DRAFT January 24, 2002, 3:11pm} \)
If \( C_{33} = 0 \) there is a control condition such that the third layer is stagnant,

\[
C_{33} = \left( \frac{1}{2} \frac{1}{2} + (\frac{1}{2} + \frac{1}{2}) \frac{1}{2} \frac{1}{2} F_1^2 + \frac{1}{2} \frac{1}{2} \left( \frac{1}{2} + \frac{1}{2} \right) \frac{1}{2} \frac{1}{2} F_2^2 \right)
\]

that is:

\[
\begin{align*}
8 & = \frac{3}{g} \frac{u_2^2}{1 \frac{1}{4} \frac{1}{8} \frac{1}{8} \frac{1}{4} c H_1} + \frac{3}{g} \frac{u_1^2}{1 \frac{1}{8} \frac{1}{8} \frac{1}{4} \frac{1}{4} c H_2} + \frac{3}{g} \frac{u_3^2}{1 \frac{1}{4} \frac{1}{8} \frac{1}{8} \frac{1}{4} c H_3} \\
& + \frac{3}{g} \frac{u_3^3 u_2^2}{1 \frac{1}{4} \frac{1}{8} \frac{1}{8} \frac{1}{4} c H_4 c H_2 c H_3} = 1;
\end{align*}
\]

Remembering that, if this control is active, the third layer is stagnant, though the control condition is only on the first two layers we can write:

\[
G = \frac{3}{g} \frac{u_2^2}{1 \frac{1}{4} \frac{1}{8} \frac{1}{8} \frac{1}{4} c H_1} + \frac{3}{g} \frac{u_1^2}{1 \frac{1}{8} \frac{1}{8} \frac{1}{4} \frac{1}{4} c H_2} + \frac{3}{g} \frac{u_3^2}{1 \frac{1}{4} \frac{1}{8} \frac{1}{8} \frac{1}{4} c H_3} = 1; \quad \text{with (A27)}
\]

\[
F_1^2 = \frac{3}{g} \frac{u_1^2}{1 \frac{1}{4} \frac{1}{8} \frac{1}{8} \frac{1}{4} c H_1} ; \quad F_2^2 = \frac{3}{g} \frac{u_3^2}{1 \frac{1}{4} \frac{1}{8} \frac{1}{8} \frac{1}{4} c H_2} \quad \text{ (A28)}
\]

The iterative procedure can be continued to define deeper control and imply the same Froude number definition that we get previously from the direct calculation of determinant.

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Figure captions:

Figure 1. Chart of the Strait of Gibraltar, adapted from Armi and Farmer (1988), showing the principal geographic features referred to in the text. Locations of current meter mooring (solid square) deployed during the Gibraltar Experiment (October 1985 to October 1986) are also shown. Areas deeper than 400 m are shaded.

Figure 2. Orthogonal curvilinear model grid; the calculated maximum departure of the grid cells from a rectangular shape is less than $10^{-12}$.

Figure 3. Model bathymetry; contour interval is 250 m.

Figure 4. (Upper) Model bathymetry, computational grid, and transects for the presentation of model results within the Strait of Gibraltar. The gray levels indicate the water depths. The points Cm and Sp mark the points where are located Spartel Sill and Camarinal Sill respectively. (Lower) Bathymetry along the longitudinal section E.

Figure 5. Initial conditions; (a) vertical profiles of salinity (solid line) and temperature (dashed line) for the Gulf of Cadiz; (b) vertical profiles of salinity (solid line) and temperature (dashed line) for the Alboran Sea.

Figure 6. Surface and bottom distribution of salinity after 3 hours (a)-(c) and 15 hours (b)-(d). Contour intervals are 0:25 psu (a)-(b) and 0:5 psu (c)-(d).

Figure 7. The temporal evolution of the along-strait density difference between the extreme edges of the strait.

Figure 8. (Upper) The temporal evolution of the eastward (inflow) and westward (outflow) transport. (Lower) The temporal evolution of the outflow salinity transport. Both curves are computed at section C.
Figure 9. (a) Scatter plot of velocity versus salinity for the strait of Gibraltar. (b) Profile of the along-strait average velocity versus salinity. Velocities are averaged over salinity layers of thickness δ S = 0.2 psu:

Figure 10. Comparison between observed and simulated mean current for eight current meter moorings of the Gibraltar Experiment. Observed data are from Candela et al. (1990).

Figure 11. Salinity cross-strait sections at Spartel Sill (a), Camarinal Sill (b), Tarifa Narrow (c) and (d). Contour interval is 0.25 psu.

Figure 12. Velocity cross-strait sections at Spartel Sill (a), Camarinal Sill (b), Tarifa Narrow (c)-(d). Contour intervals are 10 m s⁻¹ for the upper layer and 20 m s⁻¹ (a)-(b), 10 m s⁻¹ (c)-(d) for the lower layer.

Figure 13. Contour of Potential Vorticity multiplied by 10⁶ with 0.4 m s⁻¹ s⁻¹ contour interval. Gray level indicate upper layer velocity.

Figure 14. Sketch of the Mediterranean outflow over Camarinal Sill. The white thick lines delimit outflow velocity greater than 20 cm s⁻¹. White arrow represents the Mediterranean jet path.

Figure 15. Outflow velocity current (solid line) and outflow ux [u₂h₂c y] (dotted line) through the cross-sections B₁, B₂ and B₃.

Figure 16. Along strait sections E₁ E₂ E₃ of: interface depth (a), lower layer density subtract by ½ = 1028 (b) and pressure subtract by P₀ = 2.014 £ 10⁶ (c) at 200 m depth: Vertical dotted line represent position of cross-strait section B₁, B₂ and B₃.
Figure 17. Cross-section at 200 m depth for velocity (a), salinity (b) and northward pressure gradient (c). Line with triangular mark is three grid points west of Camarinal Sill, square mark is at Camarinal and circular mark is three grid points east of Camarinal.

Figure 18. Vertical profile of terms in the cross-strait momentum tendency equation for a point located along section E (a) near the western entrance of the strait, (b) just west Camarinal Sill, (c) at Tarifa and (d) at Gibraltar.

Figure 19. Two-layer interface depth with 10 m contour intervals. Gray levels indicate bathymetry.

Figure 20. Two layers composite Froude number (gray levels), calculated using mean layer velocities. Contour lines represent the bathymetry.

Figure 21. Simulated salinity distribution along the longitudinal axis of the Strait (sec. E).

Figure 22. Salinity (a), velocity (b) and density (c) profiles for a point in the strait just east of Camarinal Sill. In (a) are also plotted the monotonic cubic polynomial (square) of salinity profile (circle) and the tangent lines (t; i; b) to the monotonic cubic polynomial.

Figure 23. Mixing layer thickness with 10 m contour interval. Grey levels indicate bathymetry.

Figure 24. Variation of the eastward (inflow) and westward (outflow) transport along the strait computed after 360 days of simulation. Arrows indicate the location of the section A, B, C and D respectively.

Figure 25. Three layers composite Froude number (gray levels), calculated using mean layer velocities. Contour lines represents (thin) the bathymetry, (bold) limit the region where $G^2 > 1$. 
Figure 26. Composite Froude number for the three layers case computed along section E using the (solid line) predicted mixed layer thickness (dashed line) increased (+20 m) mixed layer thickness and (dotted-dashed line) the decreased (-20 m) mixed layer thickness.

Figure 27. Upper (solid line) and lower (dashed line) limits for the mean cross-strait composite Froude number.